Next-to-leading virtual photon - Reggeized gluon interaction[1]

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Abstract

We present the results of the calculation of the one-loop correction to the effective vertex for the quark-antiquark pair production in collisions of the virtual photon with the Reggeized gluon. This vertex is supposed then to be used for the calculation of the virtual photon impact factor, which is extremely important, for instance, for the description of the small x deep inelastic scattering in the BFKL approach.

The vertex $\Gamma^c_{\gamma^* q\bar{q}}$ we are interested in can be obtained from the projected on the colour octet state and negative signature in the t-channel amplitude of the quark-antiquark production in the virtual photon fragmentation region at collisions of this photon with any particle. For simplicity we use the amplitude for the collision of a virtual photon γ^* with momentum p_A with a massless quark Q with momentum p_B. The Sudakov decompositions for the virtual photon polarization vector e, the produced quark momentum k_1 and the antiquark one k_2 are

$$e = e_\perp + \frac{2Q}{s}p_2, \;\; k_1 = xp_1 + \frac{\vec{k}_1^{\,2}}{sx}p_2 + k_{1\perp}, \;\; k_2 = (1-x)p_1 + \frac{\vec{k}_2^{\,2}}{s(1-x)}p_2 + k_{2\perp}, \;\; k_1^2 = k_2^2 = 0, \quad (1)$$

where

$$p_A = p_1 - \frac{Q^2}{s}p_2, \quad p_B = p_2, \quad p_1^2 = p_2^2 = 0, \quad s = 2p_1p_2 \to \infty, \quad \vec{p}^{\,2} = -p_\perp^2. \quad (2)$$

The quark-antiquark pair is produced with $x \sim 1-x \sim 1$ and limited (not growing with s) transverse momenta. The virtual photon is taken in the light-cone gauge transverse to the vector p_2. The terms in the amplitude proportional to $e_\perp$ are responsible for the transverse photon and the terms proportional to Q are related to the scalar photon. We split the one-loop correction $\Gamma^{(1)}_{\gamma^* q\bar{q}}$ into three parts according to the three types of the diagrams contributing to the amplitude $\mathcal{A}^{(8,-)}_{Q\gamma^*\to Qq\bar{q}}$ with colour octet state and negative signature in the t-channel: two-gluon exchange, one-gluon exchange and t-channel gluon self-energy diagrams. The first of these contributions to the vertex can be found from the relation

$$\Gamma^{(2g)c(1)}_{\gamma^* q\bar{q}}\frac{2s}{t}\Gamma^{c(0)}_{QQ} + \Gamma^{c(0)}_{\gamma^* q\bar{q}}\frac{2s}{t}\Gamma^{(2g)c(1)}_{QQ} = \mathcal{A}^{(2g)(8,-)(1)}_{Q\gamma^*\to Qq\bar{q}} - \Gamma^{c(0)}_{\gamma^* q\bar{q}}\frac{s}{t}\omega^{(1)}(t)\left[\ln\left(\frac{s}{-t}\right) + \ln\left(\frac{-s}{-t}\right)\right]\Gamma^{c(0)}_{QQ}, \quad (3)$$

where $t = q^2 \approx -\vec{q}^{\,2}$ is the squared momentum transfer, $\omega^{(1)}$ is the one-loop Reggeized gluon trajectory and the correction $\mathcal{A}^{(2g)(8,-)(1)}_{Q\gamma^*\to Qq\bar{q}}$ is given by the diagrams of Fig.1 with the following replacement for the colour factors of their lowest lines, in order to project on colour octet and negative signature in the t-channel

$$\left(t^b t^a\right)_{B'B} \to \frac{1}{2}\left(t^b t^a - t^a t^b\right)_{B'B} = \frac{1}{2}T^c_{ab}t^c_{B'B}. \quad (4)$$

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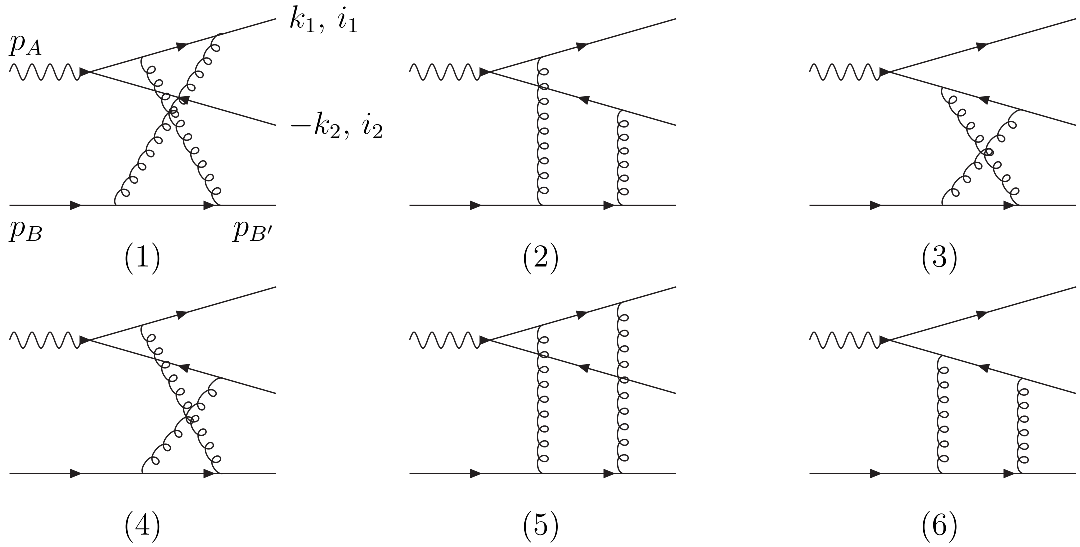

Figure 1: The two-gluon exchange one-loop diagrams contributing to the $\mathcal{A}_{Q\gamma^*\to Qq\bar{q}}$.

Fortunately, to obtain the correction $\Gamma^{(2g)c(1)}_{\gamma^* q\bar{q}}$ it is enough to calculate only two of the diagrams of Fig.1 and the correction $\Gamma^{(2g)c(1)}_{QQ}$ (see Eq.(3)), which can be found from the two-gluon contribution to the quark-quark scattering amplitude, for example. We obtain

$$\mathcal{A}^{(2g)(8,-)(1)}_{Q\gamma^*\to Qq\bar{q}} = \frac{1}{4} N t^c_{i_1 i_2} t^c_{B'B} \left\{ \left[(D_1 + D_2) - (1 \leftrightarrow 2) \right] - [s \leftrightarrow -s] \right\}, \tag{5}$$

$$\Gamma^{(2g)c(1)}_{QQ} = \frac{1}{2}\omega^{(1)}(t) \left[\frac{1}{\epsilon} + \psi(1) + \psi(1-\epsilon) - 2\psi(1+\epsilon) \right] \Gamma^{c(0)}_{QQ}, \tag{6}$$

where $\epsilon = (D-4)/2$, D is the space-time dimension, D_1 and $2D_2$ in Eq.(5) are the contributions of the diagrams of Fig.1(1) and Fig.1(2) with omitted colour generators in their vertices, and $1 \leftrightarrow 2$ is the replacement quark $\leftrightarrow$ antiquark. The one-gluon correction $\Gamma^{(1g)c(1)}_{\gamma^* q\bar{q}}$ is given by the diagrams of Fig.2 with ip_2^μ/s instead of the gluon polarization vector, and again the number of diagrams to be calculated is reduced due to the $1 \leftrightarrow 2$ symmetry:

$$\Gamma^{(1g)c(1)}_{\gamma^* q\bar{q}} = N t^c_{i_1 i_2} \left\{ \left[-\frac{2C_F}{N} (R_1 + R_2) + \frac{N - 2C_F}{N} (R_3 + R_4) + R_5 + \tilde{R}_6 \right] - \left[1 \leftrightarrow 2 \right] \right\}, \tag{7}$$

where the notations $-2R_1, ..., -2R_4, 2R_5$ and $-4\tilde{R}_6$ were used for the diagrams of Figs.2(1), ... , (4), (5) and (6) respectively, with the omitted colour generators in all vertices and the gluon polarization vector equal to ip_2^μ/s. Let us note that, whereas the definition of $R_1, ..., R_4$ is unambiguous, R_5 and $\tilde{R}_6$ are not yet well defined because of the presence of three-gluon vertices in the corresponding diagrams Fig.2(5) and (6). To complete their definition we show explicitly at these diagrams momenta and vector indices for the three-gluon vertices for which the following expression must be used:

$$ig \left[-g_{\lambda\nu}(2k+q)_\mu + g_{\lambda\mu}(k+2q)_\nu + g_{\nu\mu}(k-q)_\lambda \right]. \tag{8}$$

As for the last (self energy) contribution $\Gamma^{(se)(1)}_{\gamma^* q\bar{q}}$, it is presented schematically by the diagrams of Fig.3 multiplied with $ip_2^\mu/(2s)$ and can be calculated without any difficulties: one has only to know the Born effective vertex and the one-loop gluon vacuum polarization in the Feynman gauge (we used everywhere the Feynman gauge for the gluon fields).

The calculation of all the listed diagrams is quite straightforward but, particularly in the case of diagrams R_4 and $\tilde{R}_6$ from Eq.(7), is rather long and does not give a simple short answer. For these two most complicated diagrams we use helicity representation for their

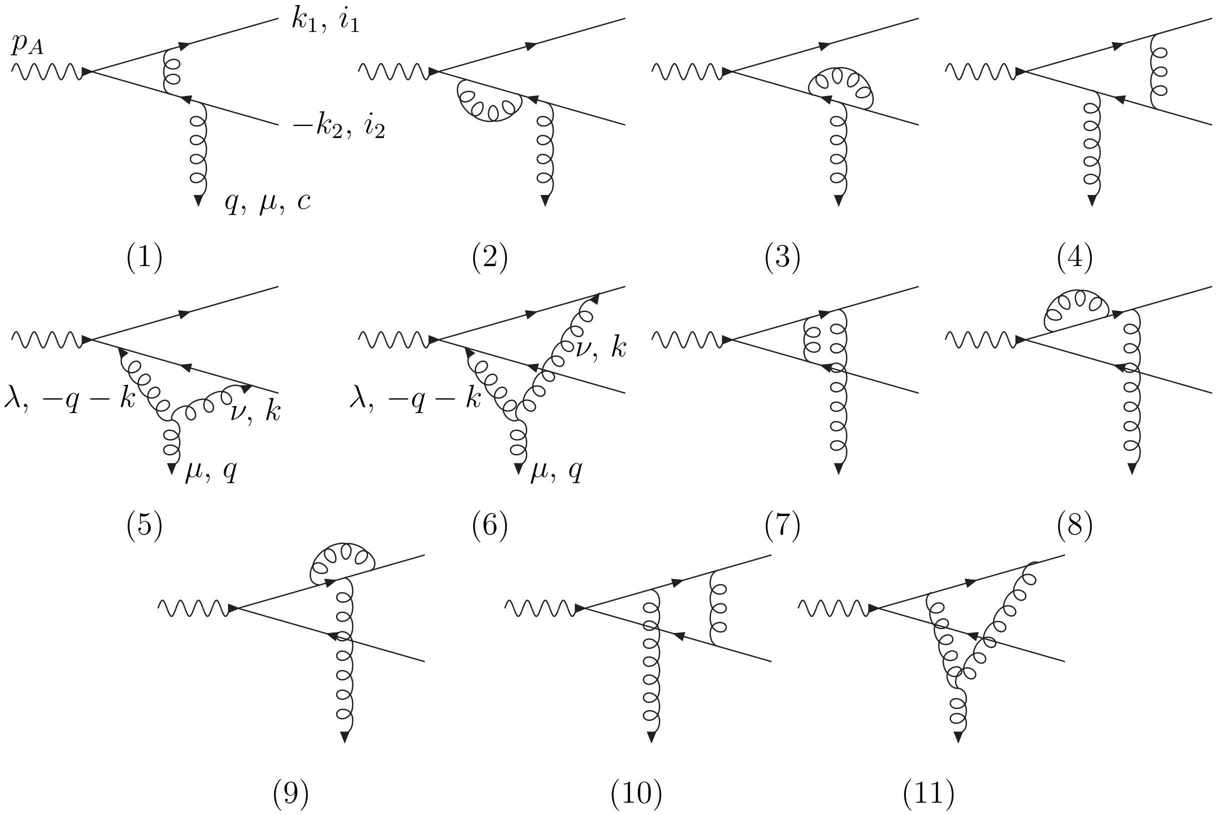

Figure 2: The diagrams corresponding to the correction $\Gamma^{(1g)c(1)}_{\gamma^* q\bar{q}}$.

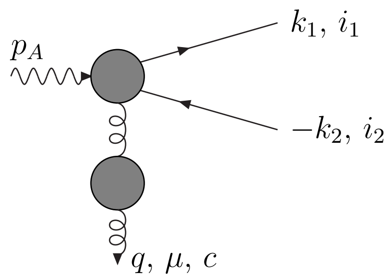

Figure 3: The diagrams corresponding to the correction $\Gamma^{(se)c(1)}_{\gamma^* q\bar{q}}$.

regular parts (finite in the physical limit $\epsilon \to 0$)

$$\hat{\rho} \equiv (v_2 \bar{u}_1) = \frac{1}{\sqrt{x(1-x)}} \frac{1}{4} \left[\left((1-x)k_1 + xk_2 - \kappa \frac{p_2}{s} \right)^{\mu} - 2i\xi e^{\mu\nu\lambda\rho} k_{2\nu} k_{1\lambda} \frac{p_{2\rho}}{s} \right] \gamma_\mu (1 - \xi\gamma_5),$$

$$e^{\mu}(\lambda) = \frac{1}{\sqrt{-2t}} \left[(\delta_{\lambda,1} + \delta_{\lambda,-1}) \left(q_\perp^\mu + 2i\lambda e^{\mu\nu\lambda\rho} q_\nu p_{1\lambda} \frac{p_{2\rho}}{s} \right) + \delta_{\lambda,0} \sqrt{-2tQ^2} \frac{2p_2^\mu}{s} \right], \quad \lambda = 0, \pm 1, \quad (9)$$

where u_1 and v_2 are bispinor wave functions of the produced quark and antiquark correspondingly, κ is the squared invariant mass of the produced pair and $\xi = \pm 1$ is the doubled helicity of the quark. For the singular parts of these two diagrams, as well as for all other diagrams, we adopt usual spinor representation to present our results. The regular parts are (we present the result for R_6 which is defined so that $2\tilde{R}_6 = R_6 - R_6(1 \leftrightarrow 2)$):

$$R_4^{(r)} = \frac{eq_f g^3}{(4\pi)^2} \frac{2}{\sqrt{2x(1-x)\vec{q}^{\,2}}} \times$$

$$\int_0^1 \int_0^1 \int_0^1 \frac{dy_1 dy_2 dy_3 y_3}{\left[-(1-y_1)y_1 y_2 y_3 t - (1-y_1)(1-y_2)y_3 t_1 + (1-y_3)\left((1-y_1)Q^2 + y_1(-\kappa - i\delta) \right) \right]^2}$$

$$\times\Big\{\Big[-(1-y_1)y_1y_2y_3t-(1-y_1)(1-y_2)y_3t_1+(1-y_3)\Big((1-y_1)Q^2-y_1\kappa\Big)\Big]\Big[((1-y_2y_3)$$
$$\times(\vec{k}_1\vec{q}+i\lambda P)-y_1y_2y_3x\vec{q}^{\,2}\Big)(1-x)\delta_{\lambda,-\xi}-y_3\left(x\vec{q}^{\,2}+\vec{k}_1\vec{q}+i\lambda P+(1-y_1)y_2(1-x)\vec{q}^{\,2}\right)$$
$$\times x\delta_{\lambda,\xi}\Big]+(1-y_3)(1-x)\kappa\Big[x\left(\sqrt{2}qQx\delta_{\lambda,0}-(\vec{k}_1\vec{q}+i\lambda P)(\delta_{\lambda,\xi}+\delta_{\lambda,-\xi})\right)+(1-y_2)\Big((\vec{k}_2\vec{q}-i\lambda P)$$
$$\times x\delta_{\lambda,\xi}+(\vec{k}_1\vec{q}+i\lambda P)(1-x)\delta_{\lambda,-\xi}-\sqrt{2}qQx(1-x)\delta_{\lambda,0}\Big)-y_1x\vec{q}^{\,2}\delta_{\lambda,-\xi}\Big]+(1-y_1)y_3\Big[(1-y_2)$$
$$\times\left(\left((1-x)\vec{q}^{\,4}+((\vec{k}_1-\vec{k}_2)\vec{q})(\vec{k}_2\vec{q}-i\lambda P)\right)x\delta_{\lambda,\xi}+(1-x)\vec{k}_1^{\,2}\vec{q}^{\,2}(\delta_{\lambda,-\xi}-\delta_{\lambda,\xi})\right)+y_1y_2x\vec{q}^{\,2}$$
$$\times\left((1-x)\vec{q}^{\,2}\delta_{\lambda,-\xi}-(\vec{k}_2\vec{q}-i\lambda P)\delta_{\lambda,\xi}\right)\Big]+(1-y_1)\left(1-y_3+(1-y_2)y_3(1-x)\right)$$
$$\times\Big[\Big(x(\vec{k}_2\vec{q}-i\xi P)-(1-y_3)(\vec{k}_1\vec{q}-i\xi P-xt_1)\Big)\sqrt{2}qQ\delta_{\lambda,0}+(1-y_2)y_3\Big(\Big(\vec{k}_1^{\,2}\vec{q}^{\,2}-xt_1$$
$$\times(\vec{k}_1\vec{q}+i\lambda P)\Big)(\delta_{\lambda,-\xi}-\delta_{\lambda,\xi})+2(\vec{k}_1\vec{k}_2-x(1-x)Q^2)(\vec{k}_1\vec{q}+i\lambda P)\delta_{\lambda,-\xi}$$
$$-(\vec{k}_1\vec{q}-i\xi P-xt_1)\sqrt{2}qQ(1-x)\delta_{\lambda,0}\Big)$$
$$+y_1y_2y_3\vec{q}^{\,2}\left((\vec{k}_1\vec{q}+i\lambda P-xt_1)(\delta_{\lambda,-\xi}-\delta_{\lambda,\xi})-2(\vec{k}_1\vec{k}_2-x(1-x)Q^2)\delta_{\lambda,\xi}\right)\Big]\Big\},\tag{10}$$
$$R_6^{(r)}=\frac{eq_fg^3}{(4\pi)^2\sqrt{2x(1-x)\vec{q}^{\,2}}}\int_0^1\int_0^1\frac{dy_1dy_2}{[(1-y_2)\left(-(1-y_1)t-y_1t_2\right)+y_2\left(-(1-y_1)t_1+y_1Q^2\right)]^2}$$
$$\times\Big(y_1\delta_{\lambda,0}\sqrt{2}qQx\Big\{\Big(x(1-x)Q^2-\vec{k}_1\vec{k}_2-i\xi P\Big)(1-3(1-x))+2(1-x)(t_2-t)\Big\}+(1-y_1)$$
$$\times\Big\{\Big[2\left((1-y_2)\left(-(1-y_1)t-y_1t_2\right)+y_2\left(-(1-y_1)t_1+y_1Q^2\right)\right)+y_1(t_2-t-\kappa)\Big]x(1-x)$$
$$\times\Big((\delta_{\lambda,-\xi}+\delta_{\lambda,\xi})(\vec{k}_1\vec{q}+i\lambda P)-\delta_{\lambda,0}\sqrt{2}qQx\Big)+\Big[2xt_1+2(1-x)t_2-2t-y_2(1-x)(t_2-t-\kappa)\Big]$$
$$\times\Big(\delta_{\lambda,0}\sqrt{2}qQx(1-x)-\delta_{\lambda,-\xi}(1-x)(\vec{k}_1\vec{q}+i\lambda P)-\delta_{\lambda,\xi}x(\vec{k}_2\vec{q}-i\lambda P)\Big)$$
$$-\Big[t\left(2x\delta_{\lambda,\xi}+3(1-x)\delta_{\lambda,-\xi}\right)+y_1x\left(t\delta_{\lambda,-\xi}-\delta_{\lambda,0}\sqrt{2}qQx\right)\Big]\Big(x(1-x)Q^2-\vec{k}_1\vec{k}_2-i\xi P\Big)$$
$$+\Big[3\left((1-x)^2t_2\delta_{\lambda,-\xi}-x^2t_1\delta_{\lambda,\xi}\right)+y_1x(1-x)\left(t_2\delta_{\lambda,-\xi}-t_1\delta_{\lambda,\xi}\right)-y_1xQ^2\delta_{\lambda,\xi}\Big](\vec{k}_1\vec{q}+i\lambda P)$$
$$+xt\Big[3(1-x)^2Q^2\delta_{\lambda,-\xi}-3\vec{k}_1^{\,2}\delta_{\lambda,\xi}+y_1xQ^2\left(\delta_{\lambda,-\xi}(1-x)+\delta_{\lambda,\xi}x\right)+2y_2(1-x)\kappa\delta_{\lambda,\xi}\Big]\Big\}\Big),\tag{11}$$
with the following notations
$$t_1=(p_A-k_1)^2=-\frac{\vec{k}_1^{\,2}+x(1-x)Q^2}{x},\quad t_2=(p_A-k_2)^2=-\frac{\vec{k}_2^{\,2}+x(1-x)Q^2}{1-x},$$
$$P=2e^{\mu\nu\lambda\rho}k_{1\mu}k_{2\nu}p_{1\lambda}\frac{p_{2\rho}}{s},\quad P^2=\vec{k}_1^{\,2}\vec{k}_2^{\,2}-(\vec{k}_1\vec{k}_2)^2.\tag{12}$$
We present the one-loop correction to the vertex $\Gamma^c_{\gamma^*q\bar{q}}$ in the form
$$\Gamma^{c(1)}_{\gamma^*q\bar{q}}=\Gamma^{(sing)c(1)}_{\gamma^*q\bar{q}}+\Gamma^{(reg)c(1)}_{\gamma^*q\bar{q}},\quad\Gamma^{(reg)c(1)}_{\gamma^*q\bar{q}}=Nt^c_{i_1i_2}\Big\{\Big[\frac{N-2C_F}{N}R_4^{(r)}+R_6^{(r)}\Big]-\Big[1\leftrightarrow2\Big]\Big\},\tag{13}$$

with the contributions of all other discussed diagrams, as well as the singular parts of R_4 and $\tilde{R}_6$, included into $\Gamma^{(sing)(1)}_{\gamma^* q\bar{q}}$:

$$\Gamma^{(sing)c(1)}_{\gamma^* q\bar{q}}\left(eq_f g^3 N t^c_{i_1 i_2}\frac{\Gamma(2-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{1}{2\epsilon}\right)^{-1} = \Bigg[\bar{u}_1\Bigg((-t)^\epsilon\left\{\frac{5}{3}-\frac{2}{3}\frac{n_f}{N}+4(1+\epsilon)\ln(1-x)+2\epsilon\Big(\frac{2}{9}\frac{n_f}{N}\right.$$
$$\left.-\frac{8}{9}-\psi'(1)\Big)\right\}\frac{\hat{\Gamma}_1}{t_1}+\frac{2C_F}{N}\frac{\hat{\Gamma}_1}{(-t_1)^{1-\epsilon}}+\frac{2C_F}{N}\int_0^1\frac{dy}{((1-y)Q^2-yt_1)^{1-\epsilon}}\Big\{(1+2\epsilon)\frac{Q^2}{t_1}\hat{\Gamma}_1+2\epsilon Q$$
$$+y\Big[(1-2\epsilon)\left(\frac{Q^2}{t_1}+1\right)\hat{\Gamma}_1+2(2-\epsilon)\left(k_{1\perp}e_\perp+xQ\right)\Big]\Big\}+\frac{1}{N}\int_0^1\frac{dy}{(-(1-y)t-yt_1)^{1-\epsilon}}$$
$$\times\Big\{2\left((1+3\epsilon)N-(1+2\epsilon)C_F\right)\frac{t}{t_1}\hat{\Gamma}_1-(2+\epsilon)N\hat{\Gamma}_1+(1+2\epsilon)N(1-x)\left(\hat{\Gamma}_1+\hat{\Gamma}_2-2q_\perp e_\perp\right)$$
$$-y^\epsilon 4(1+\epsilon)N\left(\frac{t}{t_1}-1\right)\hat{\Gamma}_1+y\Big[2\left((1-\epsilon)N-(1-2\epsilon)C_F\right)\left(\frac{t}{t_1}-1\right)\hat{\Gamma}_1+((1-2\epsilon)N$$
$$-2(2-\epsilon)C_F)\,(1-x)\left(\hat{\Gamma}_1+\hat{\Gamma}_2-2q_\perp e_\perp\right)\Big]\Big\}+\frac{N-2C_F}{N}\int_0^1\int_0^1\int_0^1 dy_1dy_2dy_3y_2^{\epsilon-1}y_3^{\epsilon+1}2\epsilon\kappa$$
$$\times\frac{y_2(1-y_2)(1-x)\left(\hat{\Gamma}_1+\hat{\Gamma}_2-2q_\perp e_\perp\right)-(1-y_2)\hat{\Gamma}_1-2(1-y_3)\left(k_{1\perp}e_\perp+xQ\right)}{\left[-(1-y_1)y_1y_2y_3t-(1-y_1)(1-y_2)y_3t_1+(1-y_3)\left((1-y_1)Q^2+y_1(-\kappa-i\delta)\right)\right]^{2-\epsilon}}$$
$$+\int_0^1\int_0^1\frac{dy_1dy_2}{\left[(1-y_2)\left(-(1-y_1)t-y_1t_2\right)+y_2\left(-(1-y_1)t_1+y_1Q^2\right)\right]^{2-\epsilon}}\Big\{y_1^{\epsilon-1}(1-y_1)y_2^{-\epsilon}$$
$$\times\left(x^\epsilon(1-x)^{-\epsilon}-2\epsilon^2\psi'(1)\right)2t\hat{\Gamma}_1+\left(y_1^\epsilon y_2^{-\epsilon}x^\epsilon(1-x)^{-\epsilon}-1\right)4(1-x)tQ+(1-y_1)\Big[2(t_2-t)\hat{\Gamma}_1$$
$$-xt_1\left(\hat{\Gamma}_1+\hat{\Gamma}_2-2q_\perp e_\perp\right)+4t(k_{1\perp}e_\perp+xQ)\Big]+y_1(1-y_1)\Big[4(t_2-t)(k_{1\perp}e_\perp+xQ)$$
$$+x(t_1+Q^2)\left(\hat{\Gamma}_1+\hat{\Gamma}_2-2q_\perp e_\perp\right)\Big]\Big\}\Bigg)\frac{\not{p}_2}{s}v_2\Bigg]-\Big[1\leftrightarrow 2\Big], \qquad (14)$$

$$\hat{\Gamma}_1=\frac{\not{k}_{1\perp}\not{e}_\perp-2x(k_{1\perp}e_\perp)+2x(1-x)Q}{x},\quad \hat{\Gamma}_2=-\frac{\not{k}_{2\perp}\not{e}_\perp-2x(k_{2\perp}e_\perp)-2x(1-x)Q}{1-x}. \qquad (15)$$

The result is valid with the accuracy up to non-vanishing in the limit $\epsilon\to 0$ terms, which is enough for the subsequent calculation of the impact factor. Although all the integrals can be calculated as expansions in ϵ with the necessary accuracy (even if such the calculation is very long, it is quite straightforward), we present our result in the integral form to have the possibility to use standard Feynman parametrization and to change orders of integration over all Feynman parameters in the calculation of the impact factor. The details of our calculation with more discussion of physics will be given elsewhere [1]. More details concerning the importance of the impact factors of colourless particles for the BFKL approach and the connection between Reggeon effective interaction vertices and impact factors can be found in [2].

References

[1] V. Fadin, D. Ivanov and M. Kotsky, to appear.

[2] The talks by V. Fadin and A. Papa in these proceedings.